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Evolufy: Enhancing Mexican Equity Portfolio Selection with Genetic Algorithms and Time Series Forecasting for Data-Driven Investment Strategies

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Resumen

Resumen de la tesis de Carlos Eduardo Sanchez Torres, presentada como requisito parcial para la obtención de la Licenciatura en Ciencias Computacionales. Ensenada, Baja California, México. Noviembre de 2023.

Evolufy: Enhancing Mexican Equity Portfolio Selection with Genetic Algorithms and Time Series Forecasting for Data-Driven Investment Strategies

Proponemos desarrollar una biblioteca para la selección de carteras de inversión que comprendan acciones mexicanas, con el objetivo de democratizar las decisiones de inversión basadas en datos. Esta herramienta aprovecha los principios de la Teoría Moderna de Portafolios, un modelo de optimización que equilibra riesgo y retorno. Emplea algoritmos genéticos para navegar eficientemente el amplio espacio de búsqueda de combinaciones de carteras potenciales. Además, los modelos de series temporales mejoran el modelo de Markowitz mediante la predicción de la eficiencia del mercado. La integración de técnicas de aprendizaje automático en la optimización de carteras ofrece estrategias de inversión más perspicaces al tener en cuenta las tendencias históricas.

Resumen aprobado por:

Dr. Luis Miguel Pellegrin Zazueta

Director de Tesis

Palabras clave: optimización multiojetivo, computación evolutiva, inversión moderna de portafolios

Abstract

We propose developing a library for selecting investment portfolios comprising Mexican shares, aiming to democratize data-driven investment decisions. This tool leverages Modern Portfolio Theory principles, an optimization model balancing risk and return. It employs genetic algorithms to navigate the vast search space of potential portfolio combinations efficiently. Additionally, time series models augment Markowitz's model by forecasting market efficiency. Integrating machine learning techniques into portfolio optimization offers more insightful investment strategies by considering historical trends.

Keywords: multi-objective problems, optimization algorithms, evolutionary computation, investment portfolio optimization, web

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Chapter 1

Introduction

Portfolio optimization is a crucial task in investment management, where the perfect balance between maximizing returns and minimizing the risk of investor users is sought. Financial institutions relentlessly aim to maximize their returns while maintaining an acceptable level of risk, making the selection of the best portfolio an essential aspect of fund management. Since [47] confirms the validity and efficiency of evolutionary algorithms, employing superior techniques will yield better-applied results, as this project intends to demonstrate. The complexity of this problem increases exponentially when considering various constraints and multiple objectives, such as risk tolerance, tax regulations, market forecasting, risk factors, and macroeconomic factors, as discussed in [11]. Moreover, the size of the search space can be enormous, especially in modern finance, where investors can choose from thousands of assets. To solve this problem, several algorithms have been developed, primarily in academia [11]. However, few of these algorithms and models use enough available information. Also, many of these algorithms do not consider realistic constraints, which can lead to unfeasible solutions in practice [3, 28, 49].

In this work, we propose developing Evolufy, an open-source library designed to solve a constrained investment portfolio optimization problem (CPOP). See more about CPOP in [2]. This is an applied project, guided by a quantitative method, and is intended for developers in the finance industry, investors, stock market speculators, managers, and other researchers interested in portfolio optimization.

In the following sections, the reader will be introduced to how to choose a portfolio based on Modern Portfolio Theory (MPT) introduced by [33, 45] and find it a solution on Multiobjective Evolutionary Algorithm (MOEA) with Time Series, and constructing a solution with Python, Darts and Genetic Algorithms. We know which are the most frequently used multiobjective evolutionary algorithms (MOEAs) and their performance metrics for measure the quality of multiobjective optimization solution by [2, 11, 28]. See Table 1.1.

Table 1.1. Types of MOEAs and Their Effectiveness in Solving CPOPs. This table shows the percentage of papers that uses the algorithm and their corresponding success scores.

MOEA type	Research Percentage (%)	Score
MOPSO		70
NSGAII	29.17	47
SPEA2	25.00	51
PESA	16.67	
SPEA	8.33	
PAES	8.33	
MOGA	4.17	
NPGA2	4.17	
IBEA	4.17	
SPEA2SDE		56
MOEADDRA		54

1.1 Problem Description and Statement

Allocate a fixed amount of capital among n assets such as stocks, funds, bonds, and so forth in order to find the efficient frontier: maximizing the expected return and minimizing the risk –the covariance matrix of returns– from recent data with a library. So, it's constrained multi-objective optimization problem.

We hypothesize that Evolufy which utilizes evolutionary algorithms, Modern Portfolio Theory, and time series, can provide an efficient and universally accessible solution for multi-objective constrained portfolio optimization. We anticipate that this approach will outperform current methods in terms of efficiency and feasibility, thereby offering end-users a superior tool for investment management.

1.2 Goals

The project aims at constructing Evolufy, an open-source software library in Python for constrained multi-objective optimization by means of evolutionary algorithms to solve a Markowitz's portfolio optimization problem with time series forecasting. It includes objectives like implementing the algorithms, benchmarking the results obtained, and suggesting improvements in space, time, and clarity about them. For that, we propose the below specific goals:

1. Develop and document a genetic algorithm and time series forecasting library for investment portfolio optimization.

2. Conduct tests and performance analysis of the implemented algorithms, using Mexican stock market data and indexes.

Chapter 2

Background

In this chapter, we present Mexican stock portfolio management and delve into financial market concepts, various investment theories, and, more specifically, Modern Portfolio Theory (MPT), which is our investing method for choosing securities. Since MPT is essentially an optimization model, we will describe what constitutes an optimization model, explain what an optimizer is, and clarify why the Genetic Algorithm fits our needs as an optimizer for finding portfolios. In a later section, we will outline what a time serie is, the autoregressive models upon which we're going to build our library, and discuss its capabilities.

The novelty of the present work lies in the selection of Mexican stocks using Modern Portfolio Theory through a novel configuration of a Genetic Algorithm with deep learning to forecast the returns. These components come together to form Evolufy, which aims to provide a computer-aided portfolio selection library for defensive investors, someone who is not willing to invest the time and effort into actively selecting securities. Specifically, our focus is on making intelligent investments that can be simplified to enable rational decision-making in the market employing Modern Portfolio Theory (MPT).

This thesis does not concern itself with the daily stock fluctuations required for technical analysis. Instead, it adheres to the definition outlined in Security Analysis, aiming to make rational decisions that, through comprehensive analysis, promise the safety of principal and an appropriate return, or equivalently, the decision minimize risk and seeks maximum utility. Modern Portfolio Theory (MPT) provides the foundation for such an analytical approach. On the other hand, if you're an aggressive investor, pursuing a Chartered Financial Analyst (CFA) designation may make sense.

2.1 Financial markets in a nutshell

$$Cash + Math \approx Finance$$

Mr. Market, who is hungry for cash.

[1] Our needs and desires are the driving forces behind our human actions. To fulfill these needs, we must organize ourselves, allocating resources and dividing labor in a manner that we refer to as the **economy**. In today's Occident, the majority of people operate within an intervened **market economy**. This system allocates resources through a complex interplay of private decisions and state intervention, using prices derived from trading as signals to guide the utilization of resources. Within a market economy, transactions occur in both product markets, which deal with manufactured goods and services, and factor markets, encompassing labor and capital. Some authors refer to the former as the *real* economy and the latter as the *paper* economy.

[1] Unless otherwise indicated, most of the definitions in this chapter are obtained from Fabozzi et al. (2010) and Brealey et al. (2007).

The paper economy provides three essential functions to the real economy: (1) it discovers the price of assets through trading and exchange; (2) it offers a mechanism to sell securities, providing some degree of liquidity to traders in the secondary market; and (3) it reduces transaction costs, including search and information costs. The former represents explicit costs, while the latter encompasses costs associated with assessing both the amount and the likelihood of the expected cash flow.

This thesis focuses specifically on common stocks, which form an essential component of financial markets. Here, stocks are considered an asset, and are thus traded in a factor market designed specifically for financial assets. The broader financial system encompasses not only these markets but also various institutions and infrastructural elements that together facilitate a comprehensive economic framework.

For instance, Alice and Bob trade assets, which are possessions that hold value to them. More precisely, Alice sell her possession because her considers more valuable Bob's cash, and Bob buys for he considers more valuable the Alice's possession than his money. These assets can be either tangible or intangible. Tangible assets have physical substance, while intangible assets consist of legal claims without physical substance but with the promise of benefit. Specifically, a financial asset (also referred to as a financial instrument or security) represents an intangible asset that carries the promise of future cash. We can identify assets by a International Securities Identification Number.

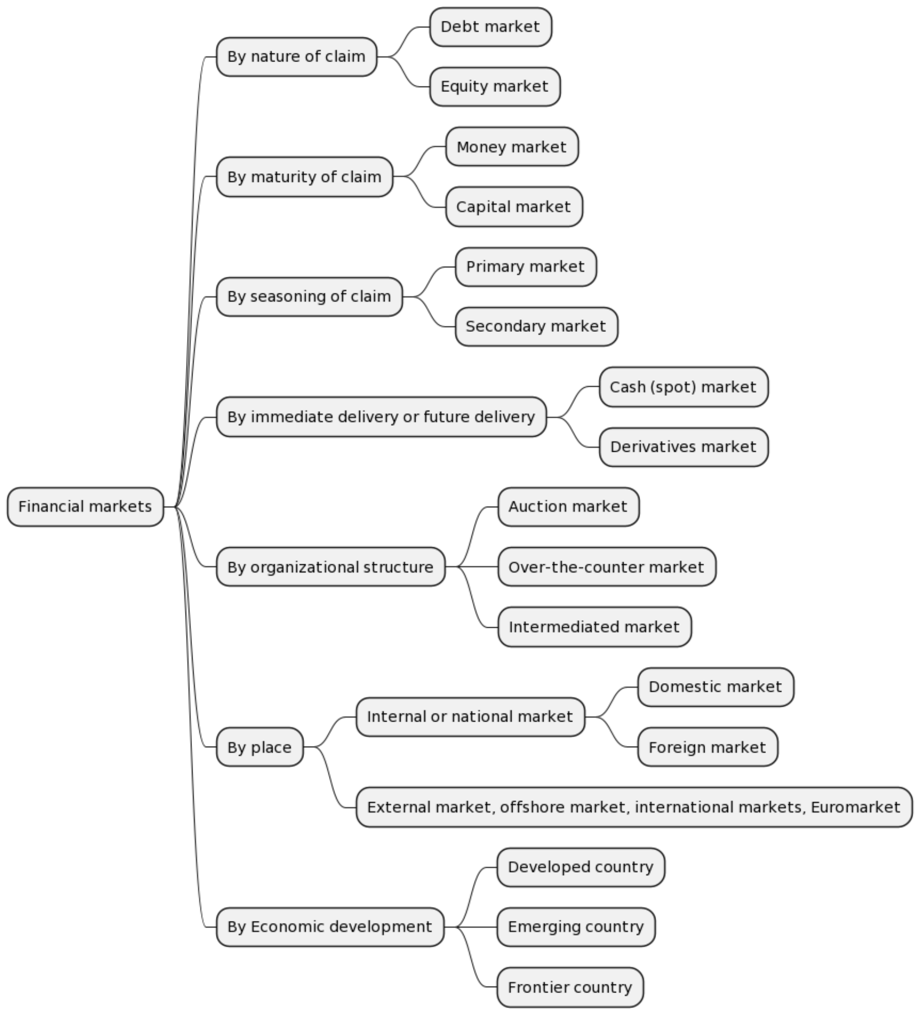
When Alice agrees to entitle future cash payments to Bob for any given reason (whether debt or equity), she becomes the **issuer** of the security, while Bob takes on the role of the **trader** (either a speculator or investor). As the trader, Bob becomes the owner of the security. Bob and Alice may agree on a fixed or varying amount. On the one hand, a debt instrument means they agree on a fixed payment. On the other hand, if they agree on an equity instrument (also referred to as a residual claim), this obliges Alice, the issuer, to pay Bob, the holder's security, an amount based on earnings. Like all human actions, Alice and Bob's choice of security depends on what fulfills their needs. When their transaction occurred, a price arose in the market, sending a signal about the value. However, a profound question arises: Did Alice and Bob exercise diligence in valuating the asset rationally? We can't guarantee that they behaved rationally, but we can assume they had incentives to make decisions that benefited themselves. Therefore, we can say that the intrinsic value differs from the price a good deal of the time, but their diligence provides us with plenty of information. Additionally, no one may uncover the intrinsic value, as doing so would imply having an extensive amount of information about the future. Consequently, a secondary function of the economy, when both parties engage in exchanges, is the redistribution of risk among the participants.

If we assume both are rational and other things held constant, Bob would make the decision to buy Alice's asset based on the present value of the expected cash flow. In other words, he analyzes the stream of payments over time, taking into account uncertainty and discount rate, to determine the instrument's present value and its expected rate of return. Uncertainty arises from various risks associated with the payments, such as inflation, default, or currency risk. However, in the real world, Bob and Alice are not alone; they coexist with a multitude of other market participants, and some of them have the power to alter the market significantly. These include a diverse range of entities such as retail investors; financial institutions like pension funds, insurance companies, mutual funds, commercial banks, and savings and loan associations; households; business entities such as corporations and partnerships; stock exchanges like NASDAQ, the New York Stock Exchange, or the Mexican Stock

Exchange; national and local governments; supranational agencies like the World Bank, European Investment Bank, and the Asian Development Bank; and regulators such as the Securities and Exchange Commission or the '*Comisión Nacional Bancaria y de Valores*'. Essentially, anyone can become an issuer or an investor, engaging in the exchange of various classes of securities and their derivatives, including common stocks, bonds, cash equivalents, real estate, commodities, private equities, hedge funds, venture capital, real assets, and currencies (e.g., crypto or forex). Figure 2.1 provides different classifications of the financial market (i.e., markets where securities are exchanged). Interestingly, colleges, including public universities, are participants in the financial market, e.g. see UABC [12, 50].

One of the biggest inquiries for traders is finding the right securities at their respective fair prices. In an efficient market, these prices reflect the aggregate information collected by all market participants. Is the existing market truly efficient? The answer to this question often depends on whom you ask and their stance on the degree of efficiency, giving rise to the term Efficient Market Hypothesis (EMH) (see [17] and [42]). This hypothesis categorizes market efficiency into three model of consistencies: weak, semi-strong, and strong. The weak form asserts that historical prices is already factored into current prices, while the semi-strong form contends that all publicly available information is reflected. The strong form goes even further, arguing that all information relevant for price formation is reviewed, even insider information. A consequence of the EMH with rational expectation is the degree to which it becomes challenging to 'beat the market' such that no investor would be able to consistently achieve higher returns than others by utilizing any type of information advantage. Related to Market Hypothesis, it is the Rational Expectation. If all participants have the same information, all participants are rational, possess

Figure 2.1. Classification of Financial Markets. This figure illustrates the various methods for classifying financial markets.



equal bargaining power, and there are no risks, Bob wouldn't be able to consistently profit. He wouldn't be able to sell his assets at a higher price because the rest would believe there's no new information to justify such a price increase. Information in this context arrives in the form of announcements, including annual earnings, stock splits, interest rates, taxes, geopolitical events, scientific insights, macroeconomic variables, and cataclysms, among others. Since this thesis pertains to computer science, we measure information in bits, which allows us to process data from different sources, provided we understand the method involved.

The one's position upon EHM and other economy premises depends on economy theories you assume such as American, Austrian, MMT, Chicago, Classical, Institutional, Keynesian, Marxist, Monetarist, Neoclassical, and New institutional. They offer us different perspectives on how economic works and how we improve its results. However, we can assume six fundamental principles of finance [29], independent of our school of thought:

- No Arbitrage Principle: There's no such thing as a free lunch.
- Non-Satiation Principle: Other things being equal, individuals prefer more money to less.
- Impatience Principle: People prefer money now rather than later.
- Risk Aversion Principle: Individuals tend to avoid risk.
- Self-Interest Principle: All agents act with their own self-interest at heart.
- Market Equilibrium Principle: Financial market prices adjust to balance supply and demand. Additionally, risk-sharing and frictions play pivotal roles in financial innovation.

Since the fundamental challenges of finance involve asset valuation and management, one can work in the financial market and achieve success using skills ranging from arithmetic to complex math. Regardless of the technique we choose, we must answer the fundamental questions: How are financial assets valued? How should financial assets be valued? How do financial markets determine asset values? How well do financial markets work? How much should I save/spend? What should I buy/sell? When should I buy/sell? How should I finance the transaction?

Addressing these challenges is particularly difficult due to the inherent involvement of time and risk. Without considering these factors, the core analytical challenges could simply be narrowed down to macroeconomic analyses. In fact, if we could effectively address these factors for the entire economy, a shift from a market-based economy to a centralized one would be conceivable. The temporal aspect pertains to disparities in cash flows across different time periods, forecasting the future, and other time series issues, such as segmentation, estimation, and explanation [5].

By trivially reducing our task to a valuation task, we invest in real assets that are worth more than they cost constrained with the available budget. We're going to explore our framework to valuate on the section 3.1. We can think of detecting intrinsic value as a time-series segmentation task. Will the market reflect the intrinsic value at some point? By definition, intrinsic value changes over time due to shifts in the economy and the flow of information. However, by applying the adaptive market hypothesis, we can envision the market attempting to approximate the intrinsic value when sufficient incentives are present. This approach assumes that the market price will eventually "correct" toward the intrinsic value, although it may deviate from it again. Borrowing terminology from distributed systems, the consistency model is eventual; in this context, how information spreads in the market and how the current market price approximates the intrinsic value can be modeled as a distributed system where agents doesn't acknowledge the same data at the same moment but someday the system is going to achieve the consistency again.We will analyze this in Section 2.3, drawing on those insights. Additionally, while other approaches have been proposed for the reader's acknowledgment, they are not the focus of this work. For example, Gómez-Águila et al. suggests using the Hurst exponent, a fractal-based analysis method, to understand the stock market's long memory.

Our second task is reducing risk. Life would be too easy if we could achieve high returns with zero risk. However, we must optimize our strategy to secure high returns with low risk. We show how in the section 2.1.2.1. After a brief discussion, we can be sure that the financial markets are the lifeblood of our economies. Now, we're going to talk about the most hectic asset in the market: stocks.

2.1.1 Common Stocks

Our earlier discourse on financial markets serves as a primer for delving into the intricacies of stocks. At the heart of stocks lie shares, each signifying a fragment of ownership in the issuing entity. Simplifying further, think of these shares as distinct slices of stock presented by a corporation. While it's customary for issuers to be public corporations gracing the listings of stock exchanges, it's not unusual for private entities to step into the limelight via an Initial Public Offering (IPO). One can gauge the magnitude of a stock through its market capitalization, a simple arithmetic of multiplying share count with its respective price. Although the broader spectrum of stocks spans both common and preferred types, our lens zooms into the latter, rooted in the foundation of a semi-strong efficient market hypothesis. Finance industry classifies stocks as equity, that is, they provide variable income, contrary to fixed income. When we trade stocks we claim them by seasoning in secondary market, except in IPO when for first time a private company offer to the public its shares. For private companies to list their shares on the BMV (stands for *Bolsa Mexicana de Valores*), certain requirements need to be met: a minimum of 200 shareholders, three consecutive years of profit, and the public must hold at least 15% of the company's shares.

It's worth noting the subtle distinctions: common stocks come with voting privileges, earning them the label of "voting shares." In contrast, their preferred counterparts often lack these rights, gaining them the designation of "ordinary shares."

Transitioning to the realm of stock exchanges, these trading hubs –be it a securities exchange or bourse– serve as bustling arenas for securities transactions. In this maze, a unique ticker symbol acts as a beacon for each issuer, a courtesy extended by the exchange. Moreover, the exchange plays gatekeeper, delineating which corporate players can step into the stock issuance arena.

Traders, in this vast ecosystem, are privy to a treasure trove of information, spanning Dividends, Intraday data, Indices, Rates, to Commodities. And as we anchor our assumptions in a semi-strong EMH, we believe that these nuggets of information seamlessly weave into the tapestry of stock prices, a theme recurrent in our prior discussions. While there's some debate about whether stock prices reflect a company's value, it's certain that owning a stock means owning an interest in a business. Strictly speaking, the stock price doesn't affect the company because it doesn't receive any returns from the secondary market, unless the board chooses to issue more shares.

What income mechanisms can one expect from investments in the Stock Exchange? The types of securities found on the Mexican Stock Exchange (BMV) are similar to those on other stock exchanges. These include stocks, debentures, government and corporate bonds, warrants, and derivatives. Similarly to other exchanges, the BMV provides typical trading facilities, making securities information accessible to the public and promoting fair market practices. With the 'Bolsa

Institucional de Valores' as its rival, our focus will be on stocks listed on the Mexican Stock Exchange.

The type of income, its frequency, its taxation, and the option for reinvestment all depend on the specific security you choose. In stocks, you might receive income through capital transactions and dividends. Capital transactions occur upon sale, while dividends are paid periodically. Taxation varies by jurisdiction; for example, in Mexico, safe harbor provisions offer tax benefits to *maquiladoras*. Dividend Reinvestment Plans allow for the automatic reinvestment of dividends.

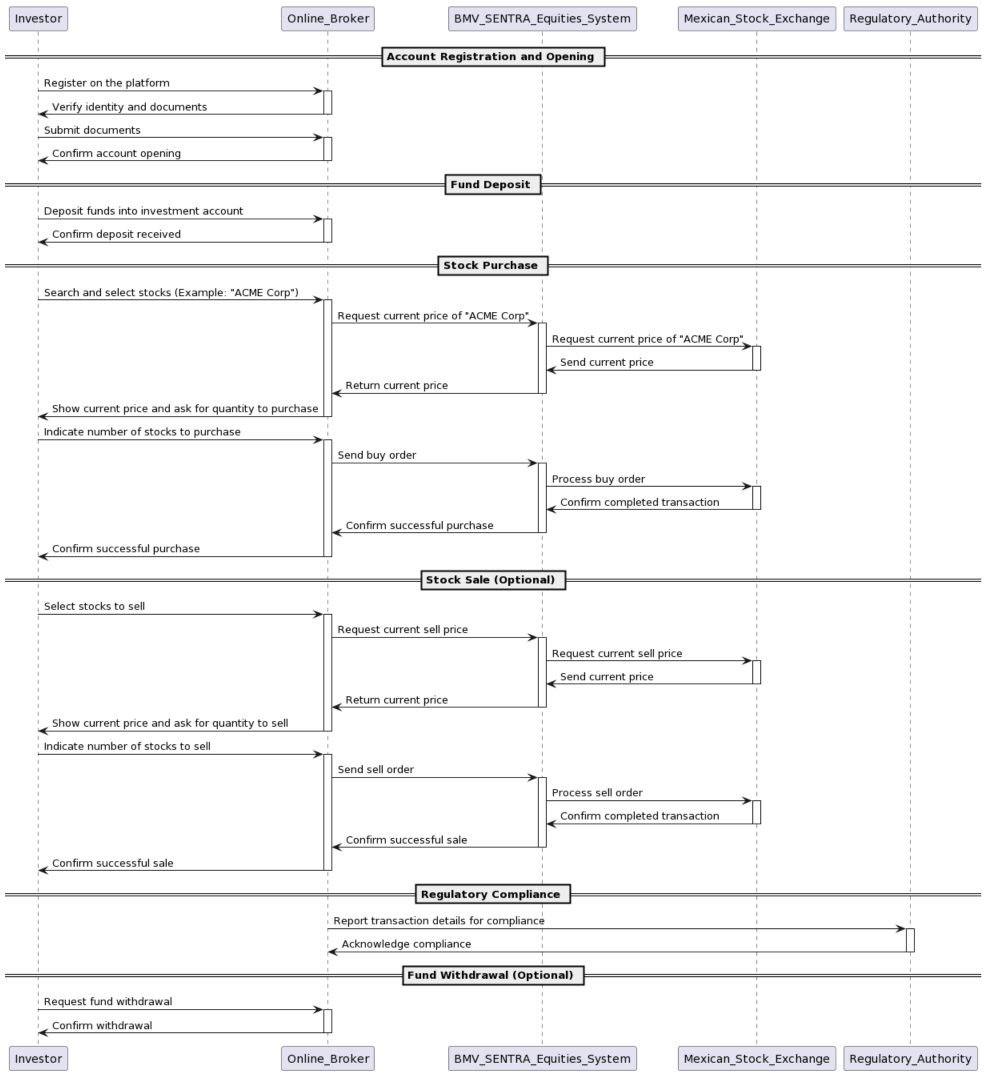
A basic rule in finance is that the valuation of an asset is equal to the present value of future cash flows. The challenge, of course, lies in the uncertainty of the future, especially since stocks offer variable returns. This means that companies provide different returns each time, necessitating effective risk management.

In the stock market, it's crucial to consider returns and cash flow policies. Over time, you might receive dividends, which represent a positive cash flow for you. Additionally, it's important to be mindful of the costs and fees. The act of buying and selling assets invariably comes with these expenses, resulting in a negative cash flow. As asset values fluctuate, periodic "rebalancing" of your portfolio may be necessary to ensure optimal allocations, potentially leading to further positive or negative cash flows.

The trading strategy informs us about the type of income, market, and policies. On the other hand, trading tactics tell us how to execute those operations in the market. In stock markets, if our income policy dictates transactions based primarily on price, that is, the return is based on the difference between the start and end of a movement. This approach is based on two scenarios: one where we believe the price is lower than it should be, and another where we think the price is higher than it should be. For the latter scenario, various tactics can be employed, such as short selling, purchasing put options, investing in inverse funds or inverse ETFs, using spreads and other option strategies, engaging in arbitrage, and pairs trading. For the former scenario, strategies include buying stocks (taking a long position), call options, leveraged ETFs, investing on margin, earning dividends, and aiming for long-term capital gains.

After choosing a trading strategy, we, as individual investors, interact with the stock exchange through both traditional and web-based brokers. Today, both types of brokers allow us to operate in the market. However, traditional brokers typically require large account sizes, whereas web-based brokers offer functionalities that enable interaction with the market through GUIs and APIs. The BMV-SENTRA Equities System facilitates brokers in conducting market operations. The BMV authorizes brokers to work with this system upon receiving a subscription payment. We describe how we trade in the market in the Figure 2.2. The BMV is a small exchange, which consists only 143 companies, compared to for example NYSE which has 5000 companies. So, applying our classification system as seen in Figure 2.1, we classify the stocks of BMV as part of the Mexican secondary equity market with variable incomes, mainly national companies which are situated in a emerging country with a Closing Cross System. In a closing cross system, the closing price of a stock is determined by taking the weighted average price of the stock based on the last trades that occur in the final moments of the trading day. This is contrary to an auction system, where buyers and sellers submit their orders, and these are matched at a specific time to determine the final price.

Figure 2.2. How to Buy Stocks on the Mexican Stock Exchange: This sequence diagram provides the answer by illustrating the roles and their interactions.



2.1.2 On Portfolio management

Res tantum valet quantum vendi potest.

Seneca

Many people aspire to accumulate wealth rapidly, but they soon discover the complications inherent in such a pursuit. So, it's wise to protect our trade adventures with the fruits of certainty so that they can withstand losses. So, we ought to allocate our resources in line with our objectives based on a framework. According to Graham and McGowan [20], these resources can be classified as investing and speculating strategies that traders employ to operate in the market. Investors aim to gain a satisfactory return through deep analysis, which must ensure the safety of the principal. In contrast, speculators involve purchasing an asset with the hope of realizing a short-term gain, based mainly on technical analysis and price changes.

It's important to remember that all security returns are contingent upon future events, and thus success largely hinges on your ability to predict these outcomes. But given the inherent uncertainties, how can we best navigate them?

Various trading theories have been devised to mitigate and manage risk, rather than eradicate it completely. Undue and undesirable risk, however, are aspects of investing that every portfolio manager seeks to control. Each theory offers unique perspectives and strategies for managing this uncertainty and achieving profit. We describe some trading theories in Table 2.1. Note that those are normative theories because it describes the trader behavior to construct a portfolio; in contract to a positive theory that describes how the trader behave.

Since all trading returns in some degree depend on future events, and those follow random walks –meaning that future steps cannot be predicted with 100% accuracy based on past history– the approaches mentioned earlier provide a framework to avoid serious blunders [32]. These approaches are mutually exclusive because their premises, reasoning, and conclusions differ in how they value assets. For instance, they presuppose different forms of the efficient-market hypothesis or apply the diversification principle as a crucial concept. Nevertheless, a portfolio management

Table 2.1. A Summary of Various Trading Theories: Different strategies have been developed for asset selection, varying in their market assumptions and decision-making approaches.

Theory Name	**Strategy**	**Works**
Buy the Rumour & Sell the Fact	Speculators buy aggressively on rumors and sell on public announcement	Brunnermeier [6]
Castle-in-the-Air Theory (Contrarian strategy)	Agents sell stocks with high past growth and high expected future growth, and buy stocks with low past growth and low expected future growth.	Lakonishok et al. [27], Keynes [25]
Constant Stock-Bond Ratio Theory	A passive investor sets a stock-/bond ratio, such as 70/30 or 80/20 (or the inverse), ignoring the current market situation.	Weston [51]
Cybernetic Analysis	Agents apply digital signal-processing techniques to forecast and explain the market.	Ehlers [14]
Deep Learning	Utilizes specific architectures (e.g., RNN or Transformer) to analyze a set of securities.	Olorunninmbe and Viktor [38]
Firm Foundation Theory (Value investing)	Investors looks for an intrinsic value by Profitability, Leverage, Liquidity, Source of Funds, and Operating Efficiency.	Graham et al. [21], Piotroski [41]
Markowitz Portfolio Selection Theory	Select a portfolio using an optimization model. In this thesis, we explain what the model consists of.	Markowitz [34]

consists the choosing the theory considering the investor's situation and its goals and make rational decisions on the market allocating investor resources to choose the right securities (i.e. a portfolio); you must also consider how investments align with your risk profile, trading time horizon, asset allocation strategy (stock vs bond, etc.), management style (active vs passive), and tax situation.

Thus, we have chosen to employ Modern Portfolio Theory because it offers optimal solutions for manager selection, guaranteed investment contracts, and value at risk assessments [15] –properties that are more challenging to replicate in other theories.

2.1.2.1 Modern Portfolio Theory

Despite criticisms of Modern Portfolio Theory (MPT) by different participants [7, 23, 37], it fits our needs as previously described. We expect that the experience from index, mutual, and pension funds, which employ MPT as their core procedure to manage portfolios, will be evidence to consider regarding its importance. For instance, Gil [18] concluded that it could serve to improve AFORES' results since SIEFORES apply passive strategies.

So, we must assist Bob, who is a rational investor and therefore seeks to minimize the level of unsystematic risk $\lambda \in \mathbb{R}^+$ given portfolio return $E[R_p]$ he is willing to accept. MPT provides a model for finding the efficient portfolio $\boldsymbol{x}$ that Bob seeks, where it defines a utility function representing the need for low risk and high returns as follows (see [34, 43, 44]):

$$\min_{\boldsymbol{x}} U(\boldsymbol{x}) = risk - returns \tag{2.1}$$

The classic MPT employs a mean-variance analysis, defining **risk** as the variance of the portfolio, denoted as $var(R_p)$, and **returns** as the expected returns, represented by $\lambda E[R_p]$. It's important to note that deriving this estimator is a distinct task, typically tackled by another method, often CAPM, which will be detailed later. Therefore, we update our utility function accordingly:

$$\min_{\boldsymbol{x}} U(\boldsymbol{x}) = risk - returns = var(R_p) - \lambda E[R_p] \tag{2.2}$$

Also the classic MPT defines the variance of portfolio as $\boldsymbol{x}^T \Sigma \boldsymbol{x}$ and the returns as $\lambda E[R_p] = \boldsymbol{x}^T \Sigma \boldsymbol{x}$ where $\boldsymbol{x}$ a vector of weights, Σ represent the covariance matrix, $\boldsymbol{R}$ be the vector the expected returns where each entry is the expected return $E[R_i]$ of the asset i. Then, $\boldsymbol{x}^T \Sigma \boldsymbol{x}$ gives the variance of portfolio return, $\boldsymbol{R}^T \boldsymbol{x}$ denotes the expected return on the portfolio such that $E[R_p] = \boldsymbol{R}^T \boldsymbol{x} = \sum x_i E[R_i]$.

In classic MPT, the variance of the portfolio is defined as $\boldsymbol{x}^T \Sigma \boldsymbol{x}$, and the returns are represented as $\lambda E[R_p] = \boldsymbol{R}^T \boldsymbol{x}$. Here, $\boldsymbol{x}$ is a vector of weights, and Σ represents the covariance matrix. $\boldsymbol{R}$ is the vector of expected returns, where each entry $E[R_i]$ corresponds to the expected return of asset i. Thus, $\boldsymbol{x}^T \Sigma \boldsymbol{x}$ calculates the variance of the portfolio's return, and $\boldsymbol{R}^T \boldsymbol{x}$ denotes the expected return on the portfolio, such that $E[R_p] = \boldsymbol{R}^T \boldsymbol{x} = \sum x_i E[R_i]$. Altogether, we derive the following optimization model:

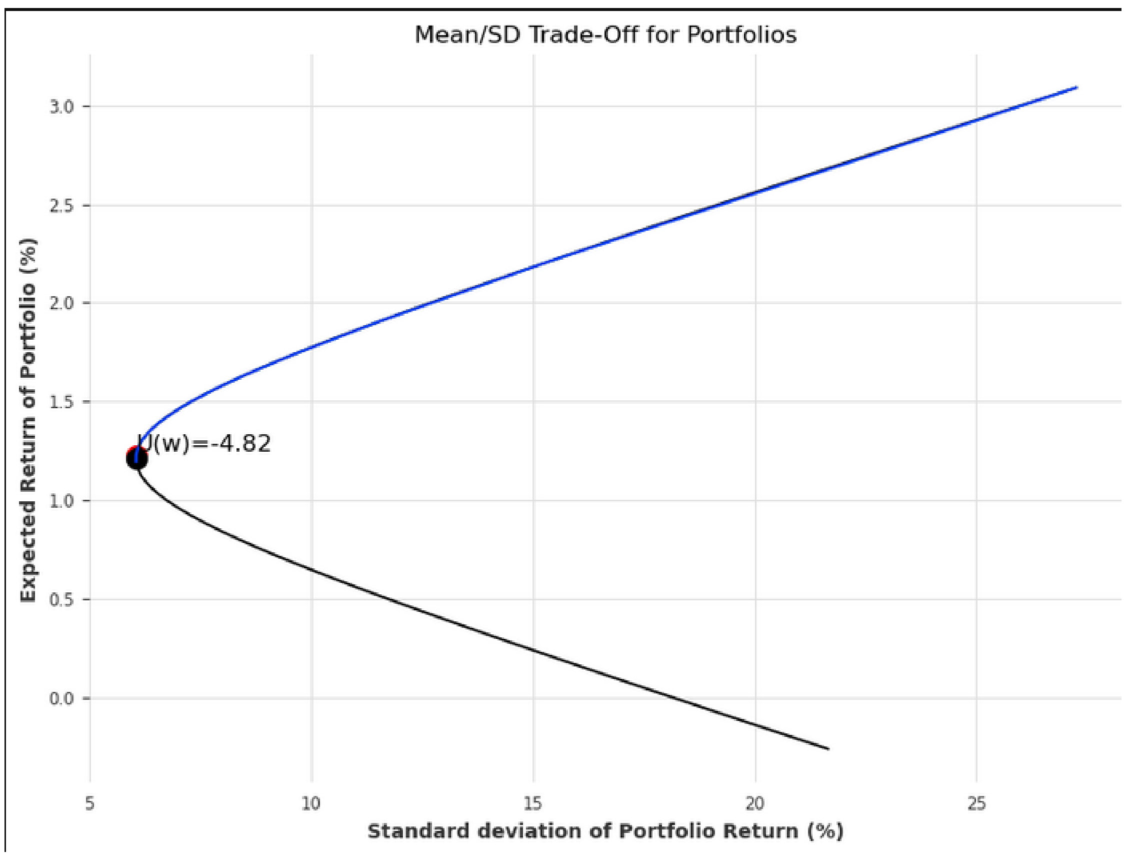
$$\min_{\boldsymbol{x}} U(\boldsymbol{x}) = risk - returns = var(R_p) - \lambda E[R_p] = \boldsymbol{x}^T \Sigma \boldsymbol{x} - \lambda \boldsymbol{R}^T \boldsymbol{x} \tag{2.3}$$

The model includes the constraint that all weights must be fully assigned, as expressed by the following equation:

$$J_{1,n} \cdot \boldsymbol{x} = 1, \boldsymbol{x} \in \mathbb{R}^n \tag{2.4}$$

Visually, as shown in Figure 2.3, the trade-off between risk and returns forms a bullet-shaped curve. In this context, we aim for the north-eastern solution on the curve, which represents low risk (with risk represented on the x-axis) and high returns (with returns represented on the y-axis).

Figure 2.3. MPT states how you minimize the risk (standard deviation) and maximize the expected return. The blue color represents the efficient frontier, which is what we're looking for. In particular, $U(w)$ is the best value for the risk $\lambda = 1$.



When you have the weak constraint $\boldsymbol{x} \in \mathbb{R}^n$ you allow short selling. When you have a stronger policy, for instance, $\boldsymbol{x} \in [0, \infty)^n$ you don't allow that behavior.

Samuelson and Nordhaus [46] approximates risk as the variability of an entity (e.g. market, or asset), in particular MPT defines risk as securities covariance $Cov(R_i, R_j)$, so Σ matrix entries are the covariance to each other:

$$\Sigma_{n\times n} = \begin{bmatrix} Cov(R_1, R_1) & Cov(R_1, R_2) & \cdots & Cov(R1, R_n) \\ Cov(R_2, R_1) & Cov(R_2, R_2) & \cdots & Cov(R_2, R_n) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(R_n, R_1) & Cov(R_n, R_2) & \cdots & Cov(R_n, R_n) \end{bmatrix} \tag{2.5}$$

$$= \begin{bmatrix} var(R_1, R_1) & Cov(R_1, R_2) & \cdots & Cov(R1, R_n) \\ Cov(R_2, R_1) & var(R_2, R_2) & \cdots & Cov(R_2, R_n) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(R_n, R_1) & Cov(R_n, R_2) & \cdots & var(R_n, R_n) \end{bmatrix} \tag{2.6}$$

In the context of MPT, we could estimate the portfolio expected returns $\boldsymbol{R}$ based on all available information in the market, which could encompasses various features of a time series such as news sentimental analysis, macroeconomics variables and company financial statements. The income policy dictates that the program should rebalance the portfolio every Δt time to incorporate the most recent closing prices. In doing so, he plans to buy and sell shares based on changes in the optimized portfolio. Additionally, if the value of his portfolio exceeds a predetermined threshold, he will sell $C\%$ of the assets.

When he rebalances his portfolio by assigning new weights, he will maintain a **long position** for asset i if its weight x_i is greater than 0. Conversely, he will opt for **short selling** if the weight of asset i is less than 0. A long position refers to the purchase of an asset with the expectation that its price will rise over time. When an investor goes "long" on an asset, they are essentially expressing a bullish outlook on its future price movement. Short selling, often termed "shorting," involves an investor borrowing shares of a stock from a broker to sell them in the open market. The objective is to buy these shares back at a lower price later, capitalizing on the stock's price reduction [5].

This thesis assists to Bob in determining the returns $\boldsymbol{R}$ by valuating assets using time series and finding an optimums portfolio with optimization algorithms[2].

2.2 Optimization models and their Algorithms

Optimization has a long history, dating back millennia. Figures such as Plato (427-347 BCE) and Aristotle (384-322 BCE) employed optimization to identify the best societal structure. More recently, George Dantzig (1914-2005) used it for resource allocation during World War II. In this work, we will focus on multi-objective optimization models from a numerical perspective, culminating in an analysis of a genetic algorithm approach to quickly find suitable solutions.

Like all effective models, optimization models offer a reliable way to conceptualize a problem. If we encounter a problem that involves finding either the best or the worst solution, it's likely that we can represent this problem with an optimization model and then apply an appropriate algorithm, as suggested by Guttag [22].

In general, the fundamental optimization problem consists of two parts: an objective function that is to be either maximized or minimized, and a set of constraints (which could possibly be empty). This can be formally written as follows:

[2] We use Jupyter Notebook to plot the figure.

$$\text{minimize } f(\mathbf{x}) \tag{2.7}$$

$$\text{subject to } \mathbf{x} \in \chi \tag{2.8}$$

where $\mathbf{x}$ is a design point, χ is the feasible set and it can be discrete or continuous, and f is the objective function. Among all points in the feasible set χ, the $\mathbf{x}$ that minimizes the objective function is called a solution or minimizer. A particular solution is denoted as $\mathbf{x}^*$. As we work with n-dimensional space, $\mathbf{x}$ is a vector and it's written as usual:

$$\mathbf{x} = [x_1, x_2, x_3, ..., x_i, ..., x_n] \tag{2.9}$$

where x_i is called decision variable or design variable.

Easily a minimization problem can be replaced by maximization problem and vice versa [26]. From

$$\text{maximize } f(\mathbf{x}) \text{ subject to } \mathbf{x} \in \chi \tag{2.10}$$

to

$$\text{minimize } - f(\mathbf{x}) \text{ subject to } \mathbf{x} \in \chi \tag{2.11}$$

But what exactly is a feasible set? A feasible set is defined by all constraints, with each constraint limiting the set of possible solutions. These constraints are expressed using the symbols $\leq$, $\geq$, or $=$ since we are working within the realm of numerical optimization. Consequently, our problem aligns with the definition of the General Multiobjective Optimization Problem provided by Coello et al. [11]:

$$F(\mathbf{x}) = (f_1(\mathbf{x}), ..., f_k(\mathbf{x})) \tag{2.12}$$

subject to

$$g_i(\mathbf{x}) \leq 0, i = 1, ..., p \tag{2.13}$$

$$h_j(\mathbf{x}) = 0, j = 1, ..., q \tag{2.14}$$

and our goal is to find global minimizer, a particular $\mathbf{x}^*$ such that $f(\mathbf{x}^*) \leq f(\mathbf{x})$.

However, some multi-objective problems present a tradeoff among costs, performance, and time, making it unclear how to prioritize constraints. When searching for a solution to such problems, we employ Pareto optimality, which represents a balance among tradeoff objectives. Subsequently, we will describe domination, a criterion used to compare two solutions. Domination is a relationship such that $\mathbf{x}$ dominates $\mathbf{x}'$ iff $f_i(\mathbf{x}) \leq f_i(\mathbf{x}')$ for all $i \in \{1, ..., m\}$ and $f_i(\mathbf{x}) < f_i(\mathbf{x}')$ for some i. In simple terms, $\mathbf{x}$ is better than $\mathbf{x}'$. The non-dominated set consists of points such that no element in the set dominates the others in the criterion space. This set is referred to as the Pareto frontier or Efficient frontier. The criterion space, denoted as χ, is the image of Υ through f. It is also sometimes referred to as the objective function space.

2.2.1 Non-dominated set generator

A subproblem involves generating a non-dominated set without a criterion space to illuminate algorithm behavior and identify the correct efficient frontier for result comparison. We have constructed a tool by applying monotonic decreasing functions and simplex spaces, as noted in the footnote[3], to display various efficient frontiers, as shown in the following Figures: 2.4, 2.5, 2.6, 2.7, and 2.8.

Figure 2.4. Synthetic Efficient Frontier generated with $Cos(\sum a_i x^i)$.

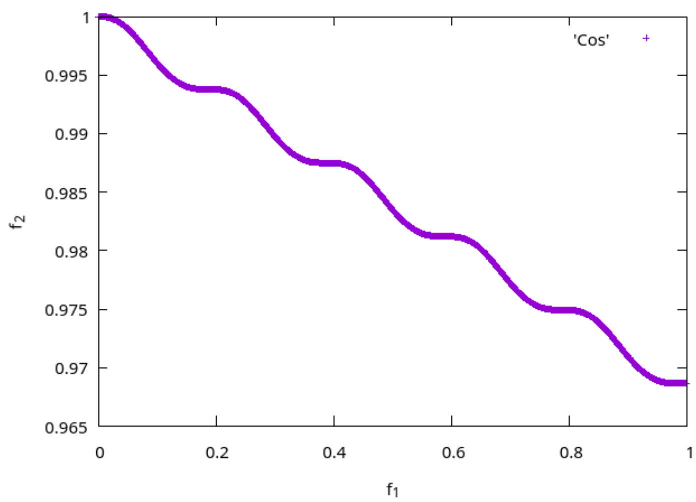
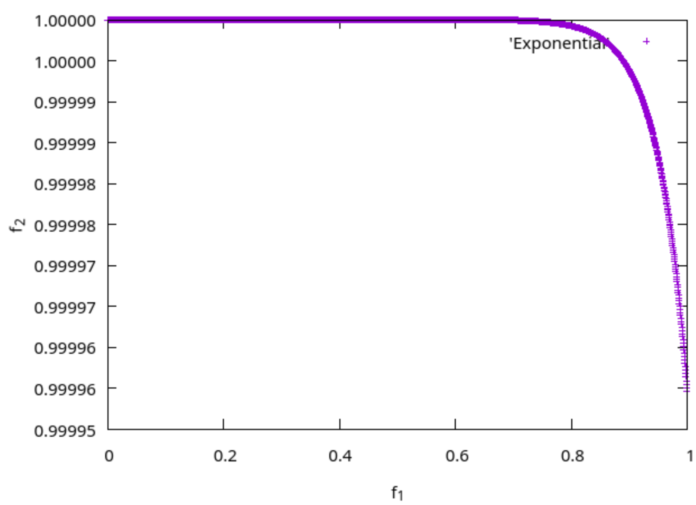


Figure 2.5. Synthetic Efficient Frontier generated with $-e^{(\sum a_i x^i)}$.



[3] Non-dominated set generator CLI

Figure 2.6. Synthetic Efficient Frontier generated with the Dirichlet distribution $Dir(1)$.

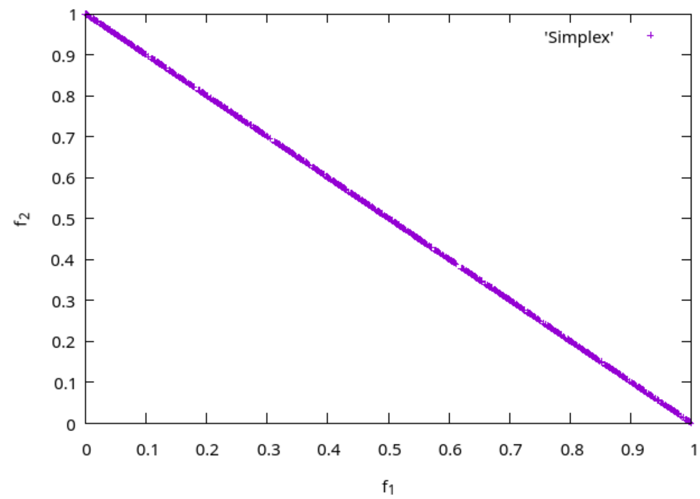
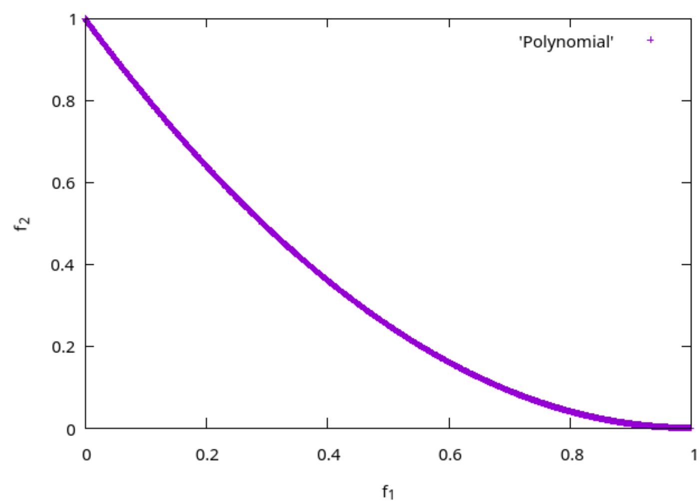


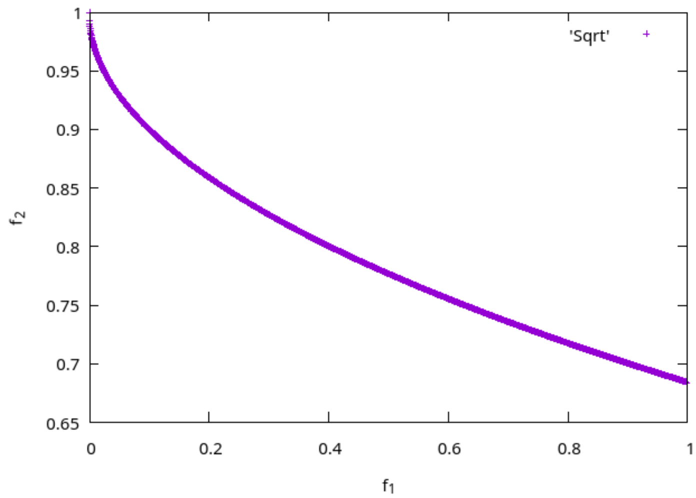
Figure 2.7. Synthetic Efficient Frontier generated with $\sum a_i x^i$, .



2.2.2 How can we solve multi-objective problems?

Optimization problems can be solved using enumerative, deterministic, or stochastic techniques, with genetic algorithms falling into the stochastic category. These heuristic algorithms are our primary focus of study. A non-exhaustive list of optimization

Figure 2.8. Synthetic Efficient Frontier generated with $-\sqrt{\sum a_i x^i}$.



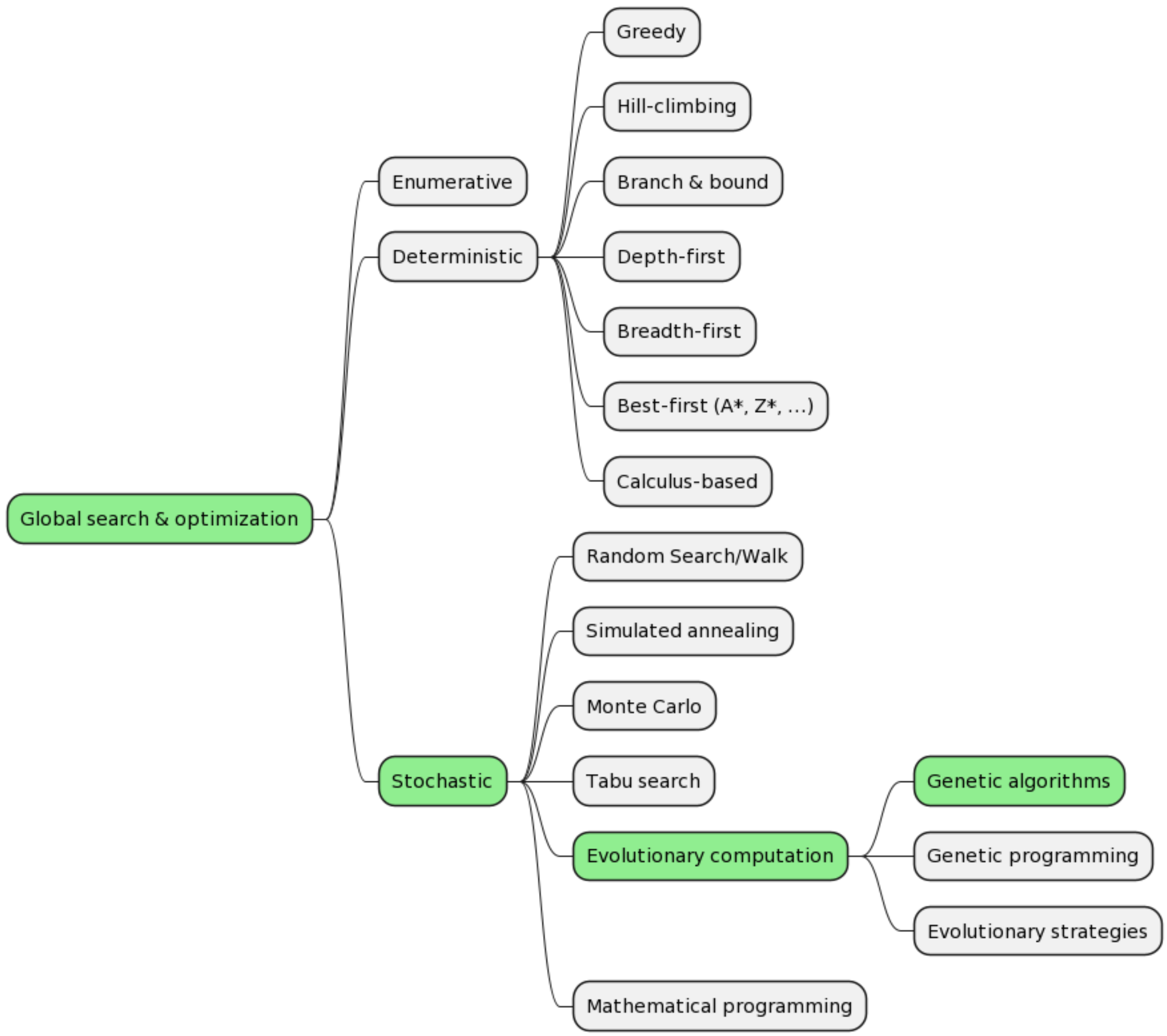
techniques can be found in Figure 2.9. The types of Multi-Objective Evolutionary Algorithms (MOEAs) and their performance scores for solving CPOP are presented in Table 1.1. Performance is measured using different metrics, as outlined in Table 2.2, with hypervolume being the most popular. Hernández-Gómez [24] developed a fast, unpublished C framework for calculating these metrics.

The reader will appreciate that the fundamental model of Modern Portfolio Theory (MPT) is a quadratic programming problem that can be solved using various

Table 2.2. Performance metrics for indicating the quality of a non-dominated approximation set

Metric
Hypervolume (HV)
Generational distance (GD)
Inverted generational distance (IGD)
Averaged Hausdorff distance (Δp)
Spread metric Δ
Spacing metric (S)
Coverage of two sets (C)
Riesz s-energy

Figure 2.9. Different global optimization approaches are illustrated, with the green path representing the algorithm used in this thesis.



methods, such as Lagrange multipliers or interior-point methods. However, our focus will be on genetic algorithms. This is because when realistic constraints are applied, the constrained investment portfolio optimization problem (CPOP) becomes a quadratic mixed-integer problem (QMIP), which is NP-hard, as detailed in [28]. Despite the principle that "there ain't no such thing as a free lunch," as mentioned in

[52], choosing the right algorithm to outperform others, as discussed in [2], is critical.

2.2.3 Genetic algorithms

Coello et al. [11] defines a genetic algorithm as a bio-inspired algorithm, using evolution as its guiding metaphor. This algorithm is situated within the broader realm of global optimization strategies, particularly in the stochastic and evolutionary domains. The categorization of genetic algorithms, along with other approaches, is illustrated in Figure 2.9. An outline is provided below for further clarity.

Algorithm 1 Basic Genetic Algorithm

```
procedure GeneticAlgorithm
    population = initialize-population-randomly()
while not reaches termination criterion()
    parents=select-parents-for-the-next-generation-with-its-fitness(population)
    children = crossover(parents)
    population = mutate(children)

    return bestIndividual(population)
end procedure
```

As outlined, the genetic algorithm comprises a population (a generalized composite data structure as illustrated in Figure 2.10). This population undergoes changes through a series of strategies, including parent selection, crossover, and mutation.

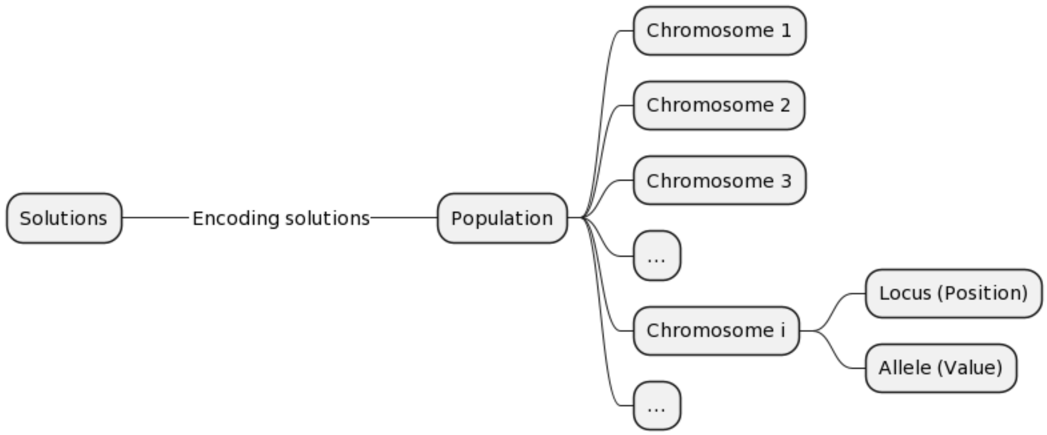
Specifically, the genetic algorithm performs the following operations on the population:

INITIALIZE-POPULATION makes a random population from design space and encodes it to binary, real, permutation, or tree.

THE TERMINATION CRITERION IS NOT TRUE uses it to determine when the algorithm ends. Perhaps, the easiest way is a generation limit.

SELECT-PARENTS-FOR-THE-NEXT-GENERATION evaluates a decoding population

Figure 2.10. In genetic algorithms, solutions are encoded as a population, which consists of chromosomes, each containing specific loci and alleles.



on the objective function, after that procedure applies a fitness function, and then it makes a mating pool with parents by the canonical selection, roulette wheel selection, or other. When the procedure always pass the best parent is called elitism.

CROSSOVER combines parents from mate pool to make offsprings. Some crossover schemes are single-point crossover, two-point crossover, uniform crossover.

MUTATE allows new traits exploration. Some mutation schemes over offsprings are flipping each bit with small rate for bit-valued chromosomes and Gaussian mutation for real-valued chromosomes.

2.2.3.1 Worked example

A worked example is useful to understand optimization problems and genetic algorithms. In genetic algorithms with elitism, each generation is closer to global minimum than previous generations how you can appreciate on the Figure 2.11. For instance, given the next function:

$$F(x, y) = min(x^2 + y^2) \tag{2.15}$$

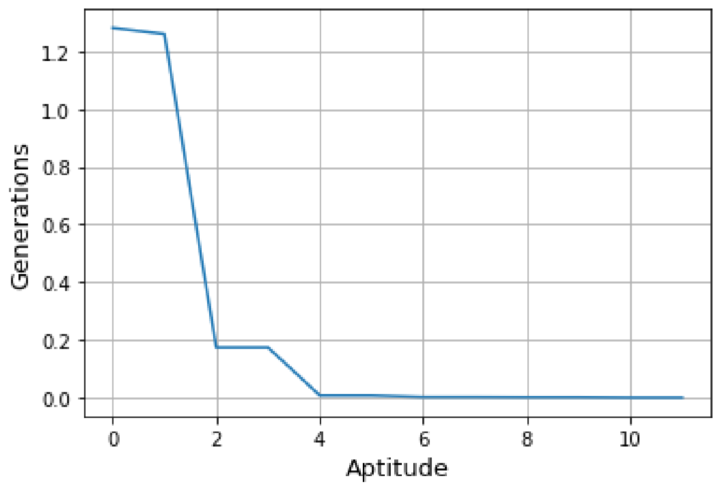
subject to

$$-5.14 \leq x \leq 5.14 \tag{2.16}$$

$$-5.14 \leq y \leq 5.14 \tag{2.17}$$

You can check out our genetic algorithm implementation on GitHub[4].

Figure 2.11. The aptitude chart displays the best fitness value for the ith generation.



[4] https://gist.github.com/sanchezcarlosjr/dc500b87169f1f0be17158ecb376e377

2.3 Time series

Viewing a time series as a stochastic process provides a framework to incorporate uncertainty and randomness, and to apply probabilistic and statistical tools for analysis and forecasting [48].

A stochastic process is a sequence of random variables, $X_1, X_2, X_3, X_4, ...$ indexed by a set. When this set represents time, the stochastic process is referred to as a time series. This type is typical for many time series in econometrics and finance, which is the focus of this thesis. Conversely, when the index set is continuous (e.g., the set of real numbers), the stochastic process is termed a continuous-time stochastic process, exemplified by phenomena such as Brownian motion. Formally, a stochastic process $\{X(t) : t \in T\}$ is a set of random variables $X(t)$ where each of them is a real-valued random variable, and t belongs to an index set T. In this thesis, the term "stochastic process" refers to a piece of information –such as stock prices, inflation rates, etc.– that is assumed to be a discrete-time series. We employ autoregressive models to explain these time series which is a recurrence relation where the current value X_t depends on p steps into the past, $X_{t-1}, X_{t-2}, X_{t-3}, X_{t-4}, ..., X_{t-p}$ and some noise w_t, that is, a recurrence of order p that has the form $X_t = \phi(t, X_{t-1}, X_{t-2}, X_{t-3}, ..., X_{t-p}, w_t)$ for $n \geq k$, where $\phi : \mathbb{N} \times X^p \to X$ is a function that involves k consecutive elements of the sequence, for that we need k initial values. Also, we will work with multivariate time series where at each time t, there are multiple observations. It is an extension of univariate time series data, which consists of single observations recorded sequentially over equal time increments. With multivariate time series data, each time you record data, you record multiple features.

In the following subsections, we will describe some models that serve to address time series forecasting in stock markets in various ways. Each of these models offers a unique approach to time series analysis, with some (like RNN, Convnets, and N-BEATS [9]) stemming from deep learning and others (like ARIMA, ARCH and GARCH) grounded in classical econometrics. The choice of model often depends on the nature of the data, the presence of volatility clusters, and the need for interpretability. In [8] and [31] present an extensive comparison of models for forecasting time series.

2.3.1 ARIMA and other classic models

ARIMA, which stands for AutoRegressive Integrated Moving Average, is a class of models that explains a given time series based on its own past values, that is, its own lags and the lagged forecast errors. The general form of an ARIMA model can be expressed as ARIMA(p,d,q):

$$(1 - \sum_{i=1}^{p} \phi_i L^i) Y_t (1 - L)^d = (1 + \sum_{j=1}^{q} \theta_j L^j) \epsilon_t \tag{2.18}$$

where p is the order of the autoregressive (AR) model (number of time lags), d is the degree of differencing (the number of times the data have had past values subtracted), q is the order of the moving average (MA) model, ϕ_i are the parameters of the AR part, θ_j are the parameters of the MA part, L is the lag operator, Y_t represents the time series, and ϵ_t is the error term at time t. The AR (autoregressive) part $(1 - \sum_{i=1}^{p} \phi_i L^i) Y_t$ represents the autoregressive part of the model, which uses past values of the series as predictors, the I (Integrated) part $(1 - L)^d$ represents the order of differencing applied to the time series to make it stationary (constant mean and variance over time), and MA (Moving Average) part $(1+\sum_{j=1}^{q}\theta_j L^j)\epsilon_t$ accounts for the moving average component, modeling the error term as a linear combination of past error terms.

Another classic model for stocks is the ARCH (Autoregressive Conditional Heteroskedasticity) model, which captures volatility clustering in financial time series data. It models the variance of the current error term or innovation as a function of the actual sizes of the error terms from previous time periods. An extension of the ARCH model, GARCH (Generalized Autoregressive Conditional Heteroskedasticity) includes lagged values of both the series' variance and the squared observation. It can model changing variances over time and is especially useful for financial time series where volatility clustering is common.

2.3.2 Deep learning approaches

1D Convolution Neural Network. ConvNets preserve the spatial structure inherent to a task by extracting features from data through filters. While densely connected layers excel at pinpointing global patterns, ConvNets are specifically designed to focus on local nuances. The basis for this distinction lies in their learning methodology: during the learning phase, ConvNets meticulously create filters to target distinct elements of the input data. Utilizing convolution operations, they achieve translation invariance, identify hierarchical patterns, and extract filters from the input feature map, ensuring uniform transformations across each one. Consequently, the defining features of a convolution layer are the size and number of its filters. Notably, a 1D ConvNet designed for time series analysis necessitates a three-dimensional input shaped as [samples, time steps, features].

RNN. An RNN (Recurrent Neural Network) is engineered to identify patterns in sequential data, such as time series or natural language. Its architecture includes loops that enable the persistence and transfer of information from one step of the sequence to the next. This makes RNNs suitable for a range of sequence-related tasks, including time series forecasting. However, a significant challenge with traditional RNNs is the vanishing gradient problem. During training, as the gradients of the loss function are backpropagated through the network, they can become increasingly small, effectively disappearing. This phenomenon makes it difficult for the RNN to learn and adjust the weights effectively, particularly in cases of long sequences, as the gradients provide the necessary information for weight adjustments during learning. To overcome this limitation, variants like LSTM (Long Short-Term Memory) and GRU (Gated Recurrent Unit) networks have been developed. These architectures introduce mechanisms like gates and memory cells, which help to maintain and regulate the flow of gradients. LSTMs, for example, have a memory cell that can maintain information over long sequences, and gates that control the flow of information into and out of the cell, thus addressing the issue of vanishing gradients by ensuring that the network can retain and access important information even over long sequences. Similarly, GRUs simplify the gating mechanism but still effectively address the vanishing gradient problem. These advancements have made RNNs more efficient and robust for tasks involving long or complex sequential data.

N-BEATS. N-BEATS (Neural Basis Expansion Analysis for Time Series) represents a recent development in time series forecasting, notable for its interpretability and reliance on neural networks. This model stands out by employing a basis expansion approach to decompose the input series. This process involves breaking down the time series into simpler, fundamental components, which are then processed through a series of stacked fully connected feed-forward neural networks.

Chapter 3

Methodology

In this chapter, we delve into our asset valuation framework –a set of assumptions, software configurations, and limitations– which forecasts stock prices without assuming a normal distribution, instead relying on a time series model based on Deep Learning. We also examine experimental settings aimed at constructing an efficient portfolio, including the configuration of a genetic algorithm and adaptations to MPT.

Subsequently, equipped with the robust model outlined in previous sections, we are well-positioned to design a language that effectively bridges the relevant domain with our software, Evolufy.

In the next section, we will conduct various validations on concrete experimental setups and test the configurations and hypotheses presented in this section.

3.1 On theory of value: Asset valuation

Money does not measure value ...

The valuation is subjective.

From Socialism: An Economic and Sociological Analysis, by Ludwig von Mises, pp. 113–22.

The experimental settings of MPT are based on the Adaptive Efficient Market Hypothesis [30], which resembles the Semi-Strong Efficient Market Hypothesis while also accounting for human nature under stressful conditions. Specifically, it assumes that the human investors are rational, possess the same information, and adapt to business conditions most of the time, except when irrational emotional stress impacts them.

We assume that returns, denoted as $\boldsymbol{R}$, do not follow a Gaussian distribution and are independent and identically distributed random variables. Indeed, our first experiment in the following section (4.1) will explain why. However, our model continues to neglect considerations of taxes and transaction costs.

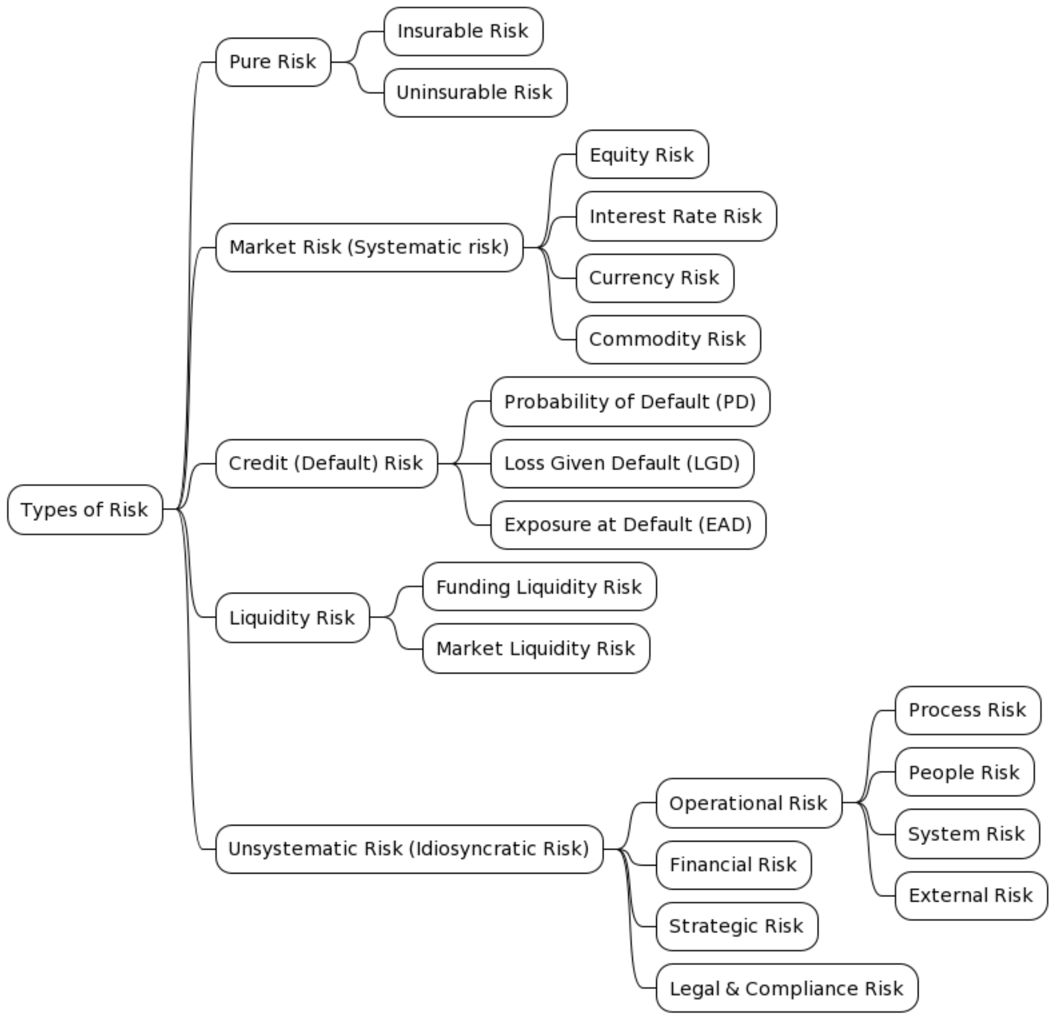
We relax these assumptions by interpreting returns as signals and, more broadly, as time series. We then apply mathematical frameworks such as signal processing, random walks, and forecasting models like ARIMA, N-BEATS, and RNN.

We seek the intrinsic value while acknowledging that the market sometimes acts irrationally. We subscribe to the Adaptive Semi-Strong Market Hypothesis with eventual and causal consistency, implying that the market will act rationally at some point in the future. Therefore, we can segment our time series to determine the intrinsic value with a confidence interval at some future point. Before proceeding, we need to establish some definitions. First, in accordance with Lo [29], we define an asset as a sequence of cash flows $CF \in \mathbb{R}$, that is, $asset = [CF_{t_0}, CF_{t_1}, CF_{t_2}, ...]$. Other way to look it as it is a discrete time serie $asset(t) = CF(t)$, $t \geq 0$. We must consider t_0 is the present time and we ignore the past to know the assets.

We convey that risk is the potential for experiencing adverse effects resulting from a given discrete set of events, which is a definition based in [40] and [13]. Analytically, risk is represented and quantified through a function $f(\boldsymbol{p}, \boldsymbol{x})$, where $\boldsymbol{p}$ is a vector of probabilities $[p_1, p_2, ..., p_n]$ associated with each adverse event in the set, and $\boldsymbol{x}$ is a corresponding vector of magnitudes $[x_1, x_2, ..., x_n]$ representing the potential impact of these events. Each element p_i in vector $\boldsymbol{p}$ indicates the probability of the ith adverse event occurring, while each element x_i in vector $\boldsymbol{x}$ signifies the magnitude of the potential impact of the ith event. The overall risk associated with the set of events is then calculated using the function, with $risk = f(\boldsymbol{p}, \boldsymbol{x})$. The specific form of function f depends on the context in which risk is assessed and the method of risk assessment being employed. On the another hand, uncertainty occurs when we cannot state a probability of distribution. A classic classification of risk is shown in Figure 3.1. Our work ignores uncertainty and focuses on market risk and unsystematic risk, measured through their volatility, i.e., covariance, standard deviation. Other metrics have been proposed to measure risk, including value at risk, semivariance, Sharpe ratio, entropic risk, downside risk, Sortino ratio, and Calmar ratio [44].

The intrinsic value is same that finding the time serie $price_{IV}(t) = V(CF(t))$ where V is the function we valuate the asset in time t given the available information. In change, the market price and random walk $price_M(t)$ is the consensus of value $V(CF(t))$ between two parties the buyer and seller with the State always be-

Figure 3.1. Risk Classification: In portfolio management, various risks are present. Some are measurable, while others are not.



ing implied. Indeed, the brokers show us the last transaction, that is, they shows us $price_M(t)$. The hypothesis mentioned before implies that in some point in the future $price_{IV}(t) = price_M(t)$ during some period of time from t_0 to t_n, so Bob will able to beat the market if he knows how. Stronger it is our assumption in both rationalism, information availability and consistency, larger span of time (Δt) because it reflects the overall market acts the best way. In the case of stocks, the cashflow is provided by the dividend $D(t)$, we ignore gain from prices difference because we consider the intrinsic value of company doesn't depend on how we evaluate the company, and the real value to investors are dividends. So, $price_{IV}(t) = V(D(t)) = \sum_{u=0}^{\infty} \frac{D(u+t)}{(1+r(u+t))^u}$ where $D \geq 0$ is a random variable in time t, $r(t)$ is the risk-adjusted discount rate for cashflow at t which it is a random variable too include the inflation, cost opportunity, taxes, currencies, term structure of interest rates, forecasting cashflows, and risk adjustments. Besides, we need an estimator P that approximates $price_{IV}$ and accounts for the difference from the actual value, which is challenging to determine.

Then, we define the time series $R \in \mathbb{R}$, representing the return at time t. These returns serve as the input for the MPT optimization model:

$$R(t) = \frac{D(t) + P(t) - P(t-1)}{P(t-1)} \tag{3.1}$$

Now, the task at hand is reduced to find either $D(t)$ and $r(t)$. However, they are complicated to estimate because all human available information is involved, and how all we know, it changes over time. What does mean available in our case? It depends on what you think the value is related to the asset and your assumptions in Efficient Market. Indeed, a trivial way to know what those values is by asking to the market.

The price believed to be going to happen is the expected value $E[P]$ of an underlying random variable P. However, P is conditioned, so $E[P|I]$ where is the available information. Also, we must consider the time serie setting of P such that it is a stochastic process indexed by time $E(P_{t+h}|I_t)$ where the price P for h period P_{t+h} is ahead to the available information I at t. A terminology for the whole population is the mean or expected returns are $\mu = E[P]$, $\sigma^2 = E[P]$, $\sigma = \sqrt{\sigma^2}$.

A forecast or prediction is a guess of unknown outcome p of an random variable P. We can forecast single points $\hat{p}$, confidence intervals, or the probability density function of P as $\hat{f}_P$.

Under Square loss, an unconditionally optical forecast of p is $E(P)$ and if we have conditions either $E[P|I]$ and $E(P_{t+h}|I_t)$. However, the expected value is not generally optimal under other relevant loss functions as absolute loss, quantile loss, and others. So other parameters of the probabilistic density function are optimal.

Besides, we have the sample (the imperfect shadow of population), the past the returns $\hat{R}$, with sample estimators $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T}\hat{R}$, $\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(\hat{R}-\hat{\mu})^2$, $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$.

Different models are provided to find the $R(t)$ such as CAMP, Gordon model, among others. However, due to their poor empirical performance, we have opted for Machine Learning techniques, a specifically Deep Learning, as [36] suggested, to more accurately estimate $E[R_i]$ using a Mexican dataset.

Among the models, the most famous is the Capital Asset Pricing Model (CAPM), which provides a formula $E[R_i] = R_f + \beta_i(E[R_m] - R_f)$ to calculate the expected returns $E[R_i]$ for an asset i. In this equation, R_f represents the risk-free rate of return, $\beta_i = \frac{cov(R_i, R_m)}{\sigma_m^2}$, $E[R_m]$ is the expected return on the market portfolio m, and $E[R_m] - R_f$ is the market risk premium (MRP). So, it gives us a baseline to make decisions and evaluate our models.

3.2 Forecasting time series with Darts

The challenge lies in forecasting and determining the intrinsic value while considering time and risk. For this task, we employ autoregressive models provided by the Darts library. Darts is a Python library designed for time series forecasting that offers a unified interface for working with various types of time series data. The library aims to provide a set of user-friendly yet powerful tools for forecasting, with an emphasis on ease of use, readability, and flexibility. It supports a range of forecasting models, including ARIMA, Exponential Smoothing, Facebook Prophet, RNN, LSTM, and others. Our setup utilizes the state-of-the-art model N-BEATS (Neural Basis Expansion Analysis for Time Series) [39], a fully connected feed-forward neural network architecture. N-BEATS has demonstrated the capability to outperform other state-of-the-art methods and provide interpretable outputs.

3.3 Modern Portfolio Theory changes

As defined in the section 2.1.2.1 on MPT, this traditional MPT utilizes variance *var* as a measure of risk. In this subsection, we describe an alternative model that employs *std* instead, thereby relaxing an assumption and potentially providing a more direct measure of risk:

$$\min_{\boldsymbol{x}} U(\boldsymbol{x}) = risk - returns = std(R_p) - \lambda E[R_p] = \sqrt{\boldsymbol{x}^T \Sigma \boldsymbol{x}} - \lambda \boldsymbol{R}^T \boldsymbol{x} \tag{3.2}$$

subject to:

$$|J_{1,n} \cdot \boldsymbol{x} - 1| \leq \epsilon, \boldsymbol{x} \in \mathbb{R}^n \tag{3.3}$$

with similar explanation that in section 2.1.2.1. The constraint has been adjusted to allow for the possibility of holding cash with a small value ϵ, which could be beneficial during periods of high volatility or when the market is expected to decline. Holding cash might reduce the overall risk of the portfolio.

3.4 Genetic algorithm configuration

The section on genetic algorithms, as previously described in 2.2.3, necessitates a configuration because it is fundamentally a metaheuristic algorithm. The term 'configuration' refers to specific strategies and their hyper-parameters, such as population size, mutation probability, elite ratio, selection and crossover types, the number of iterations, elitism, variable types, and the fitness function. In the context of genetic algorithms, each configuration property plays a distinct role in the evolutionary search process [26] [11]. The *population size* defines the number of individuals within each generation, providing a balance between genetic diversity and computational efficiency. *Mutation probability* dictates the chance of spontaneous genetic variation, introducing new traits and maintaining diversity to avoid local optima. The *elite ratio* specifies the proportion of the fittest individuals that are preserved, ensuring the retention of high-quality solutions. *Selection type*, particularly tournament selection, selects superior individuals from a random sample to be parents of the next generation. *Crossover type*, such as shuffle crossover, describes the genetic recombination method used to produce offspring from parents. The *number of iterations*, or generations, indicates how many times the population will evolve through its genetic operations. *Elitism* is the strategy of carrying forward the best solutions, preventing regression in quality. *Variable types* refer to the nature of the variables in solutions, which could be binary, discrete, or continuous. The *fitness function* is a critical component that quantifies the optimality of solutions. The *parents portion* determines what fraction of the population is selected for reproduction, influencing the rate of convergence. Lastly, the initialization of the population with a random distribution, across a population with n dimensions each, ensuring a diverse starting point for the algorithm's search.

In this thesis, we adhere to the standard configuration provided by the *geneticalgorithm* library. Specifically, we employ elitism strategies that ensure the retention of the best member in each generation. Our configuration includes 1500 generations, a population size of 10000, a mutation probability of 0.5, an elite ratio of 0.1, a parents' portion of 0.3, a tournament selection type, a shuffle crossover type, and a normal distribution for initializing a population with dimensions $(10000, n)$. Our major contribution is the design of a fitness function that encodes the constraints, as shown below:

$$\phi(\mathbf{x}) = \begin{cases} \sqrt{\boldsymbol{x}^T \Sigma \boldsymbol{x}} - \lambda \boldsymbol{R}^T \boldsymbol{x} & \text{if } |J_{1,n} \cdot \boldsymbol{x} - 1| \leq \epsilon, \\ \Delta + \|\mathbf{x} - \mathbf{1}\|_2 & \text{otherwise.} \end{cases} \tag{3.4}$$

We initiate our exploration with the framework outlined by [10] and [44], which instructs us on the incorporation of constraints into expanded fitness functions. Our method employs an exterior static penalty function, beginning from a random (potentially infeasible) starting point, and progressing toward the feasible region. This is accomplished with a penalty factor Δ that remains constant across generations plus a distance function $\|\mathbf{x} - \mathbf{1}\|_2$ to feasible region. Our fitness function operates as an unconstrained genetic algorithm when the individual is within the feasible zone; otherwise, it discourages solutions that deviate significantly from the feasible region.

3.5 Software domain

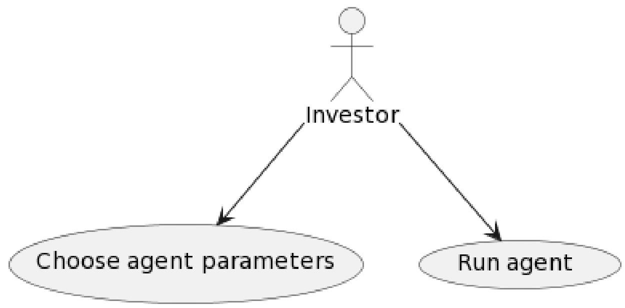
Our software, developed using the object-oriented paradigm in the Python language, is meticulously designed to align seamlessly with its intended domain. A primary emphasis is placed on ease of maintainability. In this section, we detail the design decisions made to achieve this goal. Our approach is rooted in several well-established principles:

- **Open-Close principle:** This principle asserts that software entities should remain open for extension but closed for modification. By adhering to this principle, we ensure our software can adapt to changing requirements without the need for significant alterations to the existing codebase.
- **Explicit interfaces:** This principle emphasizes the importance of clear and well-defined interfaces, ensuring components of our software interact in predictable and consistent manners. This reduces ambiguities and potential errors.
- **Small interfaces:** Advocating for concise and focused interfaces, this principle enhances their understandability and ease of implementation.

These principles, among others, draw inspiration from the foundational work of Meyer [35].

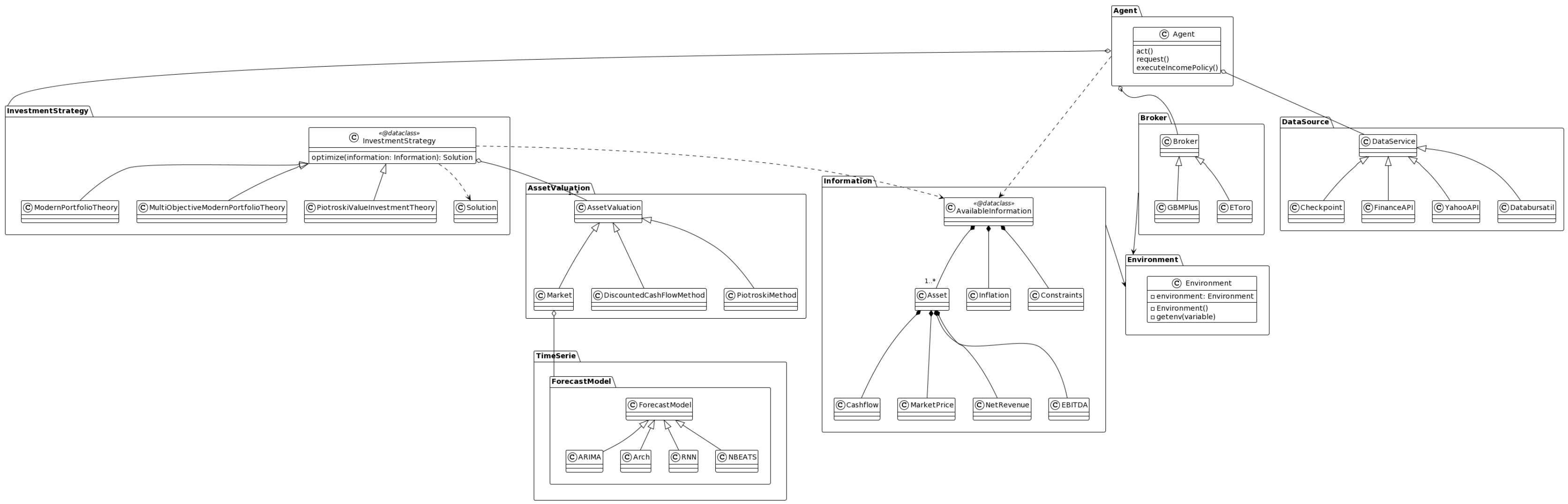
Following our initial discussion, we further explore our use cases and object-oriented models, placing a special emphasis on the design of an entity that mirrors the behavior of an investor agent. This design is illustrated in the class UML diagram 3.3 and in the use cases 3.2 that represents different aspects of Evolufy.

Figure 3.2. Use cases



The investor agent is envisioned as a virtual embodiment of an investor, tailored based on user-defined parameters. This entity is engineered to autonomously make investment decisions, leveraging predefined criteria, comprehensive data, and machine learning algorithms. Notably, the agent sources its data from platforms such as YahooFinanceAPI and DataBursatilAPI, converting this data into actionable objects that signify asset indicators, inflation rates, and other pertinent market information. Subsequently, we employ time-series models to project the intrinsic asset value and then implement the investment strategy delineated in this thesis. The final phase involves rebalancing within the actual market through a legitimate broker.

Figure 3.3. Evolufy Domain Design



Chapter 4

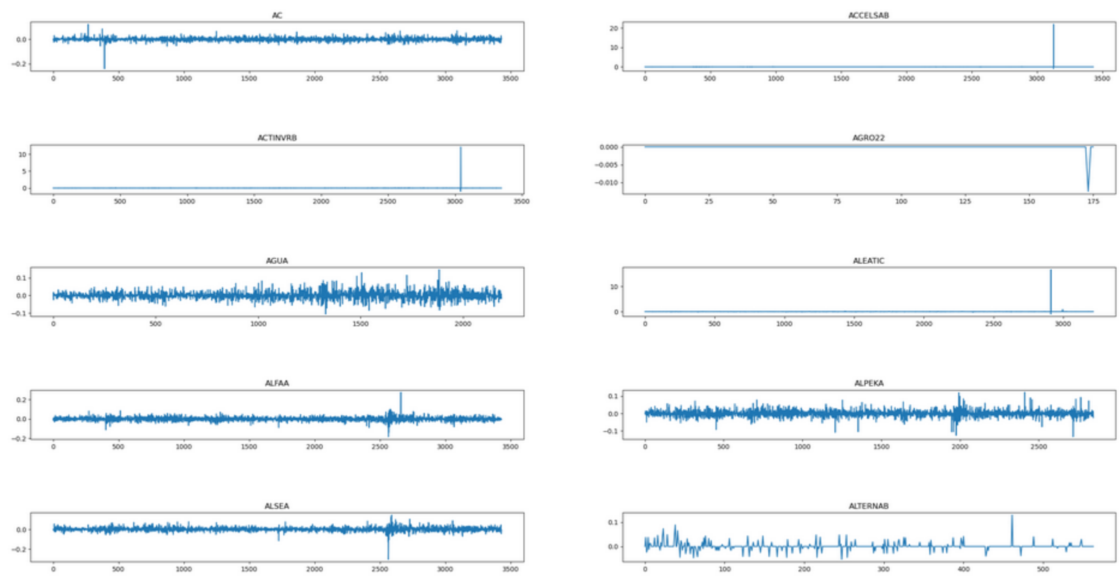
Experiments and Results

In this chapter, we critically examine the assumption that stocks follow a normal distribution, focusing on the Mexican stock market. We explore three distinct experimental settings aimed at creating an efficient portfolio for Bob, building upon the groundwork established in the previous chapter. We then present the results from these experiments, providing insights into the mechanisms he can use to navigate the market. Additionally, we will apply backtesting, a standard method for validating retrodiction datasets. This approach will enable us to test the effectiveness of our methodology against historical data. In summary, this section is dedicated to testing our methodology through various experiments.

4.1 Do daily market price changes follow a normal distribution?

Our null hypothesis asserts that the Mexican price stocks originate from a population that is normally distributed as determined by the D'Agostino's K^2 Test. If for a given stock we fail to reject $p_{value} > 0.1$, otherwise we reject the null hypothesis. We did the experiment with 100 assets[1], and the null hypothesis for all of them was rejected, so we may accept the alternative hypothesis that Mexican price stocks doesn't follow a normal distribution. Indeed, as observed in Figure 4.1, the behavior of the stock sample does not follow a normal distribution.

Figure 4.1. Mexican Daily Market Price Returns Featuring Select Stocks



4.2 Applying Fisher transformation into Mexican price stocks

Ehlers [14] proposes applying the Fisher transformation to price changes in order to normalize the data. Specifically, he states that it is possible to transform a dataset with an unknown probability distribution into a Normal distribution with mean 0 and standard deviation 1.

[1] https://github.com/sanchezcarlosjr/evolufy

Let *data* be a dataset that we wish to transform. We define the transformation as follows:

1. $data'$ is the reshaped data vector, such that $data' \in \mathbb{R}^{n\times 1}$.

2. The *scaler* is a Min-Max Scaler function which scales each element x of the reshaped data $data'$ according to the formula:

$$scaler(x) = (-0.99999 + (x - \min(data'))\frac{1.99998}{\max(data') - \min(data')})$$

3. $arctanh(data)$ receives the scaled *data* and reshapes values to approximate a normal distribution.

This transformation was applied under similar conditions as before, involving D'Agostino's K^2 Test with 100 stocks and a null hypothesis of $p_{value} > 0.1$. However, it led to the rejection of the null hypothesis. Therefore, we cannot conclude that stocks follow a normal distribution. Consequently, we need different assumptions, which will be presented in the next section.

4.3 Experiment - Naive Bob: He assumes market prices are sufficient.

Bob lacks the expertise to forecast $D(t)$ and $r(t)$, and he is unable to employ transformations or Gaussian guessing. However Bob can work under the assumption that the market is efficient, meaning that all participants are privy to the same information, and rational, in that each participant is capable of determining the accurate intrinsic value of an asset and acts in accordance with this valuation. He applies this assumption within a time window of 10 days, and he also assumes that the market price has already factored in future dividends, so $P_M(t) = P_{IV}(t)$ over t_0,t_1, ..., t_{10}. But Bob must aggregate the information that market prices provide as $E[P_M(t)|I]$, because the price is a consensus between two parties, but the market price aggregate the price from all participants. He doesn't assume the price follows a Normal distribution, so his task is to find the $E[P_M(t)|I]$ for each asset. As previously mentioned, given the square loss criterion, the optimal forecast for $p_M(t)$ is $E[P_M(t)|I]$. To estimate this, we train a deep learning model such that it gives us a estimator $\hat{p_M(t)}$ for each asset. So, he has a tensor shape $(time, features, samples) = (3600, 1, 100)$, so he runs up against univariate stochastic time serie. Nevertheless, Bob has chosen a single feature, which is the difference in historical closing prices, as his primary source of income comes from capital transactions rather than dividends.

Since log scale provides us symmetry and the same bounding $(t > 1)$ we employ to describe the random variable $R(t)$.

$$R(t) = log_{10}\left(\frac{P_M(t)}{P_M(t-1)}\right) = log_{10}(P_M(t)) - log_{10}(P_M(t-1)) \tag{4.1}$$

And we estimate the expected returns $E[R(t)]$ as

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} \hat{R(t)} \tag{4.2}$$

$$= \frac{1}{T}\sum_{t=1}^{T}[log_{10}(\hat{P_M(t)}) - log_{10}(P_M(\hat{t} - 1))] \tag{4.3}$$

$$= \frac{1}{T}(log_{10}(\hat{P_M(T)}) - log_{10}(\hat{P_M(0)})) \tag{4.4}$$

where $T = 10$ because it is given for the Bob's window of 10 periods.

He's going to test his idea with a 48 assets, then compare the results with a baseline, which happens to be an index in the market. If he can achieve better results than those of the market, he will have beaten the market. For instance, the USA markets have the S&P 500 and the Dow Jones Industrial Average (DJI). The Mexican market features the "Price and Quotation Index Close," provided by *Banco de México* [1]. This is a capitalization-weighted index that represents the performance of the 35 largest and most liquid stocks listed on the BMV.

This experiment evaluation involves backtesting, which entails testing our predictive model on historical data. Specifically, we train a model from the beginning of time until January 01, 2022. The agent will adapt to data from that starting point and execute our genetic algorithm and MPT version to select a portfolio with varying asset weights, while assuming fractional shares. Subsequently, we simulate 365 days of daily trading with those weights. We will then benchmark the returns of this portfolio against the IPC ˆMXX during those months, utilizing different metrics.

4.4 Results

In our backtest, we assessed the investment strategy by constructing a portfolio consisting of 48 diverse assets and we've ignored transaction costs. We then benchmarked this portfolio's performance against a baseline market index during a live test. Our investment model was developed and trained using historical data up to the year 2022. For comparative analysis, we selected the 'Price and Quotation Index Close' as our benchmark index, focusing specifically on its performance during the year 2022. To replicate the IPC ˆMXX, a passive investor can buy the ETF iShares NAFTRAC. We propose a daily rebalancing strategy using same weights determined by our configuration, which includes a genetic algorithm with a fitness function, a time series forecast using the NBEATS model, and a selection of 48 fractional shares. Pyfolio, a Python package, provides us with a variety of different benchmarks that we will need to explain.

Essentially, our strategy performs well on the in-sample data, but it does not perform well on out-of-sample data, as Table 4.1 shows. Also, it also doesn't outperform the IPC index as Table 4.2 indicates.

Table 4.1. The performance metrics of our investment strategy suggest that it exhibits lower performance in the region of interest and higher performance in previous periods. Here, "in-sample" refers to the period of time before the validation period, and "out-of-sample" pertains to the live validation. Typically, the higher the Sharpe ratio, the better.

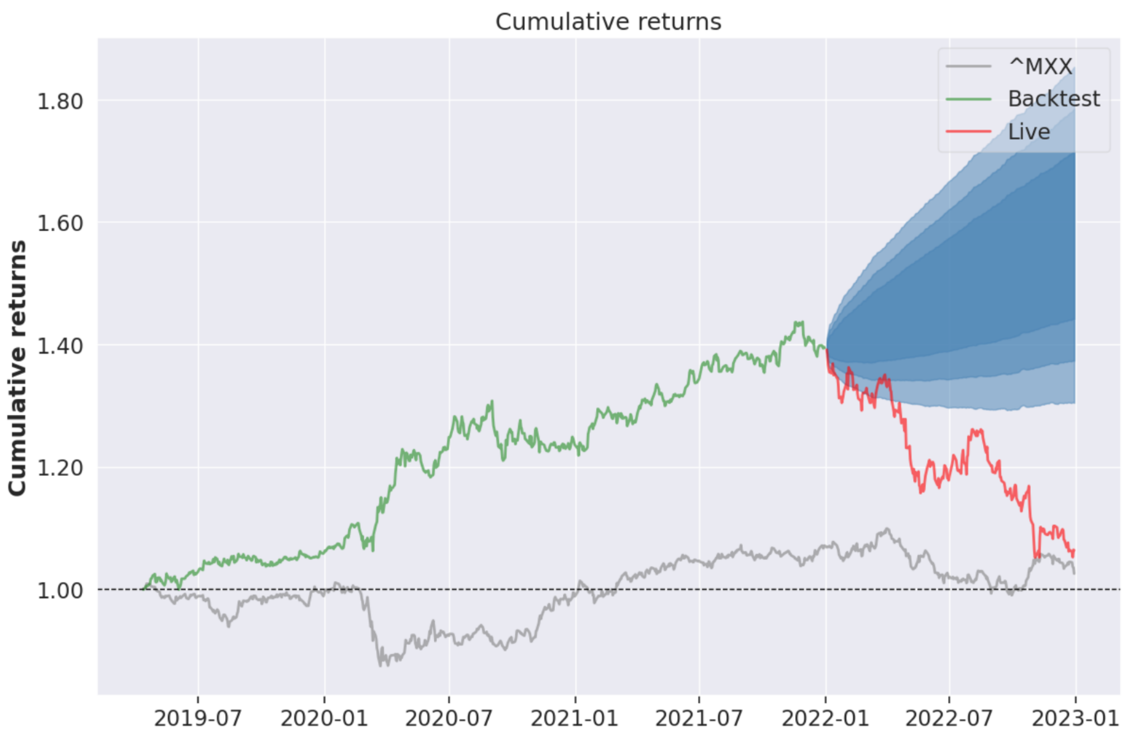
	In-sample	**Out-of-sample**	**All**
Annual return	12.5%	-23.082%	1.61%
Cumulative returns	39.42%	-23.72%	6.349%
Annual volatility	8.7%	14.037%	10.445%
Sharpe ratio	1.40	-1.80	0.21

Table 4.2. The performance metrics of a following the IPC index have better results in out-of-sample

	In-sample	**Out-of-sample**	**All**
Annual return	2.472%	-4.142%	0.657%
Cumulative returns	7.132%	-4.271%	2.556%
Annual volatility	7.748%	7.055%	7.567%
Sharpe ratio	0.35	-0.56	0.12

The Figure 4.2 below provides insight into why we outperformed in live data and outperformed the market in the backtest. Our hypothesis is that the N-Beats model may have overfit in the past, and it might not have provided sufficient valuable information for the optimization model during that time period. We propose, in future work, to increase accuracy by including experiments with more comprehensive market information such as dividends, inflation, etc. Additionally, we suggest implementing online learning, which involves retraining and optimizing the model daily to stay updated with the most recent information available.

Figure 4.2. Cumulative returns are represented by the green curve for ”in-sample” and the red curve for ”out-of-sample.” The gray curve represents the benchmark index. The blue region indicates the Bayesian prediction for the returns in backtest.



Chapter 5

Conclusions

This thesis makes a contribution to the field of financial investment by introducing a library designed to manage investment portfolios, offering the potential to enhance investor returns.

Our hypothesis is centered on the potential to improve portfolio performance through the integration of evolutionary algorithms with Modern Portfolio Theory and time series analysis. The results we have presented validate the effectiveness of our methods, demonstrating their adaptability and ability to benefit from market volatility. However, for these methods to yield better return forecasts, a profound change in risk measurement is required, such as incorporating downside risks.

Through the deployment of a well-designed library, we provide investors with the essential tools to navigate the complexities of financial markets and potentially profit from them.

Furthermore, this thesis explores the potential for developing innovative strategies that can outperform the market by leveraging 'Banking on Prices and Dividends.' It aims to harness the full spectrum of market information by incorporating multivariate time series analysis, which goes beyond models like the Fama-French three-factor model by considering a broader range of available information. Additionally, we propose, as future work, comprehensive sentiment analysis and thorough backtesting against other strategies, including the Harry Browne Permanent Portfolio, Ray Dalio's All Weather Portfolio, Intraday Trading, Value Investing, and passive investing strategies like those involving the S&P 500.

Appendix A

On scales

The return rate $R(t)$ in ratio scale gives us

$$R(t) = \frac{P_M(t) - P_M(t-1)}{P_M(t-1)} \tag{A.1}$$

We say a scale is symmetric if only if $R(t) + R(t+1) = 0$ when $P_M(t-1) - P_M(t+1) = 0$ for all P_M.

Suppose we have the a gain in time t and lost in time $t+1$ with the same absolute scale such that

$$R(t) + R(t+1) = (P_M(t) - P_M(t-1)) + (P_M(t+1) - P_M(t)) \tag{A.2}$$

$$= P_M(t-1) - P_M(t+1) = 0 \tag{A.3}$$

However, in the same conditions, the ratio scale gives us $R(t) + R(t+1) = 0$ if and only if $P_M(t) = P_M(t-1)$, so the ratio scale is asymmetric because it doesn't hold for all P_M. We prove it as follows:

$$R(t) + R(t+1) = 0 \tag{A.4}$$

$$\frac{P_M(t) - P_M(t-1)}{P_M(t-1)} + \frac{P_M(t+1) - P_M(t)}{P_M(t)} = 0 \tag{A.5}$$

$$\frac{P_M(t)}{P_M(t-1)} + \frac{P_M(t+1)}{P_M(t)} - 2 = 0 \tag{A.6}$$

Since $P_M(t-1) = P_M(t+1)$, we apply it (remember same conditions).

$$\frac{P_M(t)}{P_M(t-1)} + \frac{P_M(t-1)}{P_M(t)} - 2 = 0 \tag{A.7}$$

$$\frac{P_M^2(t) + P_M^2(t-1)}{P_M(t-1)P_M(t)} - 2 = 0 \tag{A.8}$$

$$(P_M^2(t) - P_M^2(t-1))^2 = 0 \tag{A.9}$$

and it holds when $P_M(t) = P_M(t-1)$.

Now, we need a scale that be symmetric and provide us a ratio. The log scale have came to rescue us.

$$R(t) + R(t+1) = 0 \tag{A.10}$$

$$[log_{10}(P_M(t)) - log_{10}(P_M(t-1))] + [log_{10}(P_M(t+1)) - log_{10}(P_M(t))] = 0 \tag{A.11}$$

$$log_{10}(P_M(t-1)) - log_{10}(P_M(t+1)) = 0 \tag{A.12}$$

$$log_{10}(P_M(t-1)) - log_{10}(P_M(t-1)) = 0 \tag{A.13}$$

Therefore, log scale symmetry holds for all P_M.

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